Surface Wave Propagation Through a Small Gap Between Oppositely Magnetized Ferrite Substrates

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Abstract—**This paper presents an investigation of electromagnetic wave propagation in a thin dielectric slab sandwiched between oppositely magnetized ferrite substrates. It is found that this configuration supports forward surface wave modes with a lower cutoff frequency $\omega_L = \gamma H_0$, which is smaller than the lower cutoff frequency $\omega_L' = \gamma[H_0(H_0 + 4\pi M_0)]^{1/2}$, for the surface waves in previously examined structures. Backward surface waves propagate in a broad band in the high frequency region. When the saturation magnetizations of the substrates are the same, the group delay time varies linearly with the wave frequency as well as the biasing field throughout the range of allowed modes, except near the cutoffs.**

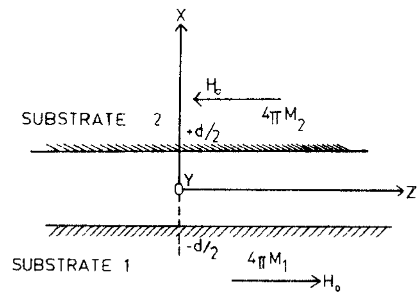

Fig. 1. The structure investigated in the problem. The substrates extend to infinity along the Y- and Z-axes

I. Introduction

In recent years, theoretical and experimental investigations have been reported on the propagation of magnetostatic surface waves in a variety of planar structures, e.g., unbacked slab [1], [2], metal-backed slab [3], [4], magnetically anisotropic slab [5], dielectric layered structure [6], layered magnetic structure [7], [8], etc. The characteristics of the surface waves are different in different structures; the waves may be highly dispersive or relatively nondispersive [6], forward or backward [7], reciprocal [1] or nonreciprocal [3], and may be allowed in narrow or broad frequency band. However, a common feature is that the lower cutoff frequency for the surface waves in all these configurations is the same [9] and is given by $\omega_L' = \gamma[H_0(H_0 + 4\pi M_0)]^{1/2}$. It follows that magnetostatic surface waves of arbitrarily small frequency can be propagated by applying a sufficiently small biasing field. However, in practice, the domain structure comes into the picture at low biasing field strengths which, in turn, leads to the absorption of the wave [10]. In fact, it is difficult to propagate low-loss magnetostatic waves below about 3 GHz [11].

Tsutsumi [7] investigated the behavior of magnetostatic surface waves propagating through an air gap between two adjacent ferrite substrates which are similarly magnetized. The dispersion relation for the surface waves propagating through a gap between oppositely magnetized ferrite substrates (Fig. 1) can be obtained, under the magnetostatic approximation, from Tsutsumi's dispersion relation [7] by making appropriate changes in the signs of the off-diagonal elements of the permeability tensors for the ferrites. Such an analysis reveals that the lower cutoff frequency is reduced to $\omega_L = \gamma H_0$. However, the magnetostatic approximation is known to be invalid near the cutoff [12], [13], and hence a rigorous electromagnetic analysis is required to confirm this result. In what follows, the propagation of electromagnetic waves (transverse to dc magnetization) in a thin dielectric slab sandwiched between oppositely magnetized, semi-infinite ferrite substrates (Fig. 1) has been investigated.

II. Theory

Maxwell's equations are separable [13] into two sets: one for E_x, E_y, and h_z (TM mode) and the other for E_z, h_x, and h_y (TE mode). The TM mode is of no interest as it does not interact with the magnetic nature of the medium [14]. The relevant components of the electric and magnetic fields, for the TE mode, in the gap and magnetic substrate regions may be written from Maxwell's equations as follows:

$$\begin{aligned} E_z^{(1)} &= \exp(\delta_1 x)\exp(-j\beta y)\\ E_z^{(g)} &= [A\exp(K_0 x) + B\exp(-K_0 x)]\exp(-j\beta y)\\ E_z^{(2)} &= D\exp(-\delta_2 x)\exp(-j\beta y) \end{aligned} \tag{1}$$

and

$$\begin{aligned} h_y^{(1)} &= \frac{jC}{\omega\mu_{\text{eff}}^{(1)}}(\beta\kappa_1/\mu_1 - \delta_1)E_z^{(1)}\\ h_y^{(g)} &= -\frac{jCK_0}{\omega}[A\exp(K_0 x) - B\exp(-K_0 x)]\\ &\quad\cdot\exp(-j\beta y)\\ h_y^{(2)} &= \frac{jC}{\omega\mu_{\text{eff}}^{(2)}}(-\beta\kappa_2/\mu_2 + \delta_2)E_z^{(2)} \end{aligned} \tag{2}$$

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where

$$\delta_i^2 = \beta^2 - \varepsilon_i \frac{\omega^2}{C^2} \mu_{\text{eff}}^{(i)}$$

$$K_0^2 = \beta^2 - \varepsilon_g \omega^2 / C^2$$

$$\mu_i = \frac{\omega_0(\omega_0 + \omega_{mi}) - \omega^2}{\omega_0^2 - \omega^2}$$

$$\kappa_i = \frac{\omega \omega_{mi}}{\omega_0^2 - \omega^2}$$

$$\mu_{\text{eff}}^{(i)} = \frac{\mu_i^2 - \kappa_i^2}{\mu_i} = \frac{(\omega_0 + \omega_{mi})^2 - \omega^2}{\omega_0(\omega_0 + \omega_{mi}) - \omega^2}$$

$$\omega_0 = \gamma H_0, \qquad \omega_{mi} = \gamma \cdot 4\pi M_i. \tag{3}$$

Here ω, H_0, γ, $4\pi M_i$, ε_i, ε_g, and C represent the wave frequency, biasing field, gyromagnetic ratio, saturation magnetization of ith substrate ($i = 1,2$), dielectric constant of ith substrate, dielectric constant of the gap material, and the speed of light in vacuum, respectively. The matching of E_z and h_y at $x = \pm d/2$, followed by elimination of constants, leads to the following dispersion relation for wave propagation in the $+Y$ direction.

$$\exp(2K_0 d) = \frac{(\mu_{\text{eff}}^{(1)} K_0 + \beta\kappa_1/\mu_1 - \delta_1)(\mu_{\text{eff}}^{(2)} K_0 + \beta\kappa_2/\mu_2 - \delta_2)}{(\mu_{\text{eff}}^{(1)} K_0 - \beta\kappa_1/\mu_1 + \delta_1)(\mu_{\text{eff}}^{(2)} K_0 - \beta\kappa_2/\mu_2 + \delta_2)}. \tag{4}$$

The signs of κ_1 and κ_2 should be reversed in order to obtain the dispersion relation for propagation in $-Y$ direction. In the case of identical substrates ($M_1 = M_2$), (4) reduces to

$$\exp(2K_0 d) = \left(\frac{\mu_{\text{eff}} K_0 + \beta\kappa/\mu - \delta}{\mu_{\text{eff}} K_0 - \beta\kappa/\mu + \delta}\right)^2. \tag{5}$$

III. Dispersion Characteristics

Surface wave modes correspond to real and positive values of δ and real K_0 in (4) and (5). In the case of identical substrates, Fig. 2 shows the dispersion curves for the surface waves propagating in the $+Y$ direction, as obtained from magnetostatic analysis as well as from (5) for $d = 0.025$ cm (Case ①) and $d = 0.1$ cm (Case ②). The biasing field strength has been shown alongside each curve. According to the magnetostatic analysis, the range of allowed surface modes is $\omega_0 < \omega < (\omega_0 + \omega_m)$; the two limits correspond to the cutoffs while resonance occurs when $\omega = \omega_0 + \frac{1}{2}\omega_m$. The surface waves are forward (backward) waves in the region below (above) the resonance. The lower cutoff frequency obtained from the rigorous analysis is $\omega_L = \omega_0$, i.e., in accordance with the magnetostatic analysis. Near the lower cutoff frequency, the departure from magnetostatic analysis is less pronounced for smaller gap widths and lower biasing field strengths; the curves almost coincide for $H_0 = 0$. The dispersion curves in the backward wave region also coincide with magnetostắtic curves but are bounded by the straight line $K_0 = 0$. Consequently, the cutoff frequency for the backward waves is less than the magnetostatic limit given by $\omega = \omega_0 + \omega_m$. The higher the dielectric constant of the gap material; the lower is the cutoff frequency for backward waves. The most significant departure from the magnetostatic analysis, as revealed by the rigorous analysis, is the occurrence of a "forbidden band" when ω is slightly smaller than ω_L', in which case μ_{eff} becomes so large that δ cannot remain real without being inconsistent with (5). This band is absent for $H_0 = 0$ for obvious reasons and is broader for larger gap widths, implying that the novel surface wave branch in the range $\omega_L \leq \omega < \omega_L'$ is significant only for sufficiently small gap widths. It is also noteworthy that when d is small and H_0 is large, the forbidden band is obtained in the high wave number region, thereby invalidating the magnetostatic approximation in this region. This is in contrast with most other situations where the magnetostatic approximation leads to correct (qualitatively, if not quantitatively) results in the high wave number region.

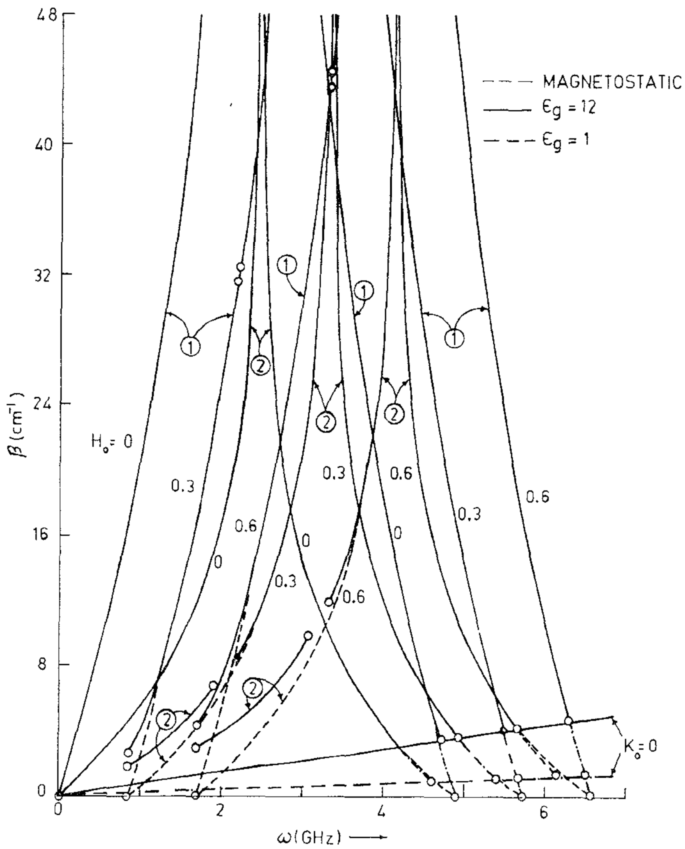

Fig. 2. Variation of the surface wave propagation constant β with frequency ω (expressed in GHz) for various biasing field strengths for $d = 0.025$ cm (Case ①) and $d = 0.1$ cm (Case ②). Other parameters are $4\pi M_0 = 1.75$ kg, $\gamma = 2.8$ MHz/Gauss and $\varepsilon_1 = \varepsilon_2 = 12$. The curves for $\varepsilon_g = 1$ (—·—·—) and $\varepsilon_g = 12$ (———) overlap except near the upper cutoff

In the region slightly away from cutoffs, where the magnetostatic approximation is valid, the group velocity is found to be

$$v_{g/d} = (\omega_0 + \tfrac{1}{2}\omega_m - \omega) \tag{6}$$

i.e., the group velocity is linearly dependent on the wave frequency and biasing field; this is important from the view point of signal processing [15].

When the two substrates have different magnetizations, the forward and backward wave regions separate out and a forbidden band is created in the range $\omega_0 + \frac{1}{2}\omega_{m_1} < \omega < \omega_0 + \frac{1}{2}\omega_{m_2}$ (assuming $M_2 > M_1$). In this case, the general characteristics of the surface waves are similar to those for identical substrates. However, the dependence of the group delay time on ω or H_0 is no longer strictly linear (results not shown).

The analysis for the case of propagation in the $-Y$ direction requires numerical investigations of (4) and (5), with the signs of κ_1 and κ_2 reversed. It is found that no surface wave modes propagate in the $-Y$ direction whether the magnetizations of the substrates are same or different. This can be understood in terms of the field displacement effect because of which the surface wave propagates in the direction of $H_0 \times \hat{n}$ [9] where $\hat{n}$ is the outward normal at the surface. It follows that in the present configuration (Fig. 1), the surface wave can propagate only in one direction, the $+Y$ direction, since both H_0 and $\hat{n}$ have opposite signs for the two substrates.

The unidirectionality of the surface waves is not inconsistent with thermodynamics, since modes other than the surface modes do propagate in the $-Y$ direction. For instance, (5) is identically satisfied for $K_0 = 0$ or $\beta = \sqrt{\varepsilon_g}\ \omega/C$. This "optical mode" is reciprocal and propagates only when $\varepsilon_g > \varepsilon_t \mu_{\text{eff}}$; this condition is ordinarily satisfied when $\omega > \omega_L'$. The transverse field distribution for this mode is evanescent in the substrates and uniform within the gap, where it resembles a plane wave. Bulk or volume modes are also allowed when δ is real and positive while K_0 is purely imaginary, which is possible only when $\beta < \sqrt{\varepsilon_g}\ \omega/C$. Under this condition, (5) is satisfied provided that $\varepsilon_g > \varepsilon_t \mu_{\text{eff}}$, i.e., for $\omega > \omega_L'$. These bidirectional bulk modes are fast and nonreciprocal. Even when $\omega_L \leq \omega < \omega_L'$, the propagation of energy in the $-Y$ direction would occur in a realistic (finite) structure, in the form of a surface wave at outer edges or as an attenuated wave with heat loss.

IV. Conclusion

It has been found that the configuration involving a thin dielectric slab sandwiched between oppositely magnetized, semi-infinite ferrite substrates can support magnetic surface waves for which the lower cutoff frequency is $\omega_L = \gamma H_0$, which is smaller than that in any other configuration investigated so far. The surface waves are forward (backward) waves in the low (high) frequency region. The group delay time varies linearly with the wave frequency and biasing field, except near the cutoff. The rigorous electromagnetic analysis reveals a small forbidden band (not obtained in the magnetostatic analysis) in the forward wave region when ω is slightly smaller than ω_L'; this is a unique situation where the magnetostatic approximation turns out to be invalid even in the high wave number region.

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